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ASPECTS REGARDING THE MATHEMATICAL MODELLING OF NON CONVENTIONAL PROCESSES OF PLASTIC DEFORMATION

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ABSTRACT: Dieless drawing of wires and bars is a non conventional process of plastic deformation which present some advantages compared to classical one. Dieless drawing is an incremental process of plastic deformation by the help of local inductive heating. In this process temperature and plastic deformation affect each other, that is why the deformation behavior is very complicated. The aim of this paper is to present some principles of mathematical modelling for the dieless drawing process. With the models which are presented it is possible to study the influence of various process parameters like: temperature, strain, strain rate, stress. Also the paper present some elements of the process control for producing variable cross-sections(cone-contour).

KEYWORDS: non-conventional, drawing, dieless, mathematical modelling

1. INTRODUCTION:

In conventional metal forming processes, large plastic deformation is given to a material using tools such as dies. Some materials present much difficulties in forming because of high strengthes or poor ductilities. In most of forming processes large frictional force acts on the interface between the material and the tool, which has a negative influence in the process. To decrease the flow stress and to increase the ductility hot working is often used, but there still remain problems caused by heat resisting tools and lubricants in the high temperatures. The non-conventional processes seem to solve these problems; they are a kind of hot working and frictionless processes. Dieless drawing is a non conventional and incremental process of plastic deformation. A particularity of that kind of process is that the deformation is continuous on short parts of the semiproduct.The result of this fact is that the deformation stress, power and energy consumed in the process are lower than in the classical processes where the deformation take place in the whole volume of material. Dieless drawing is an incremental metal forming process for plastic stretching of bars, wires, by using local induction heating. The wiredrawing die is an integral and crucial component in the conventional drawing process and much research and design work has been dedicated to producing dies with optimum die angles . The objective of the dieless process is to produce wire of equal quality to that of conventional drawing, but without the use of dies or mechanical tools. The technology of dieless drawing , relies on an increse of wire temperature to lower the yield stress of the axially loaded workpiece material and to cause local plastic deformation. The wire is passed through a controlled air flow which subsequently cools it to stop the deformation and the fracture of the material.
The desired cross-sectional area is determined by the velocities of the wire entering and exiting the deformation zone.
The main parameters of the process can be classified into:

-control parameters: temperature, drawing force, velocity
-kinematic parameters: strain, strain rate
-material properties: density, heat conductivity, specific heat capacity
-mechanical parameters: yield stress
-disturbance parameters:inhomogeneity of material properties, changes of temperature, drawing force and velocity

2. ELEMENTS OF MATHEMATICAL MODELLING

For the analyze we assume that a round bar with initial radius R_0 is drawn through a fixed inductor to final radius R. Ingoing and out going speed V_0 and V_1 and also the drawing force F are constant , so that the process is in a steady state(Fig.1).The shape of the deformation zone is described by the function R=R(X). The equivalent strain is calculated with relation 1[3].

$$\varepsilon = \ln(\frac{A_0}{A_1}) = \ln(\frac{R_0^2}{R_1^2}) = 2\ln(\frac{R_0}{R_1}) \qquad (1)$$

From relation 1 by differentiation with respect to time t , result the equivalent strain rate (2).

$$\varepsilon = \frac{d\varepsilon}{dt} = \frac{d\varepsilon}{dx}\frac{dx}{dt} = \frac{d\varepsilon}{dx}V \qquad (2)$$

where V=V(x)

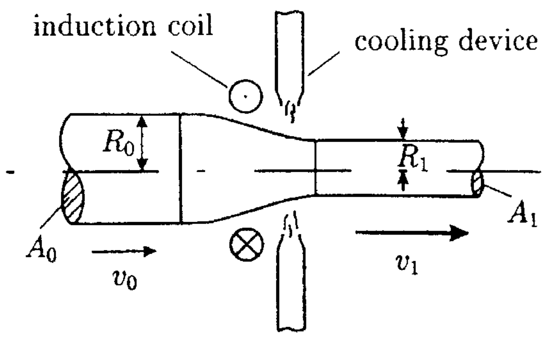

***Figure 1** Model of deformation*

Taking into account the law of volume constancy and with the notation $\frac{dR}{dx} = R'$ results equation 3.

$$\varepsilon = -\frac{2V_0R_0^2}{R^3}R' \qquad (3)$$

As the radial component of the material flow in the deformation zone is very small compared to the longitudinal one a uniaxial stress distribution σ= σ(x) can be assume.
The force equilibrium demands that the drawing force F is constant along the bar, from where result relation 4.

$$F = \sigma_0 \pi R_0^2 = \sigma \pi R^2$$

$$\sigma = \sigma_0 (\frac{R_0}{R})^2 \qquad (4)$$

Considering the plastic deformation as a creep process , it can be considered the function σ = σ(T, ε, ε) as having the shape from relation 5[3]:

$$\sigma = Ce^{-m_1 T}(\varepsilon + \varepsilon_0)^{m_2} e^{m_4/(\varepsilon+\varepsilon_0)} \varepsilon^{m_3} \qquad (5)$$

where, C, m1, m2, m3, m4 are constants of material.
For numerical evaluations it is useful to solve equation 5 for R' after introducing equation 3.

$$R' = -\frac{R^3}{2V_0 R_0^2} \left\{ \frac{\sigma \exp(m_1 T)}{C(\varepsilon + \varepsilon_0)^{m_2} \exp[m_4/(\varepsilon + \varepsilon_0)]} \right\}^{1/m_3} \qquad (6)$$

In order to include the effect of large plastic deformation , we assume that the temperature T is only a function of the longitudinal coordinate x, the temperature gradient in radial direction can be neglected.Considering a disc with infinitesimal thickness dx, cut out of the deformation zone, we can set up a balance of ingoing and outgoing heat per unit time(Fig.2).

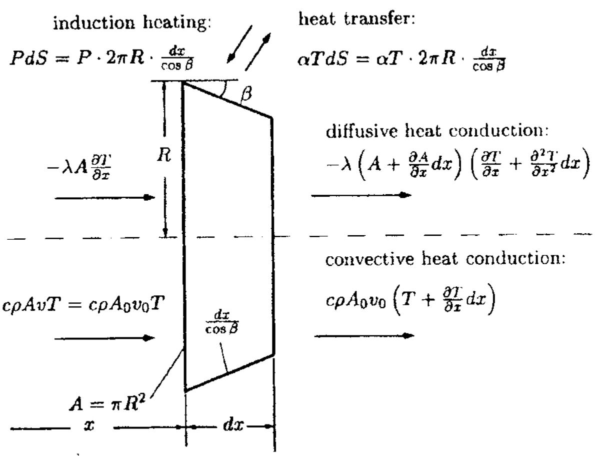

***Figure 2** Model of heat transfer*

Assuming a steady state conditions , the temperature does not depend on time and heat generation by plastic deformation can be neglected. From the heat equilibrium equation on the disc result equation 7.

$$\frac{\partial^2 T}{\partial x^2} - (\frac{2}{R} tg\beta + \frac{V_0 c\rho}{\lambda} \frac{R_0^2}{R^2}) \frac{\partial T}{\partial x} - \frac{2\alpha}{\lambda R \cos\beta} T + \frac{2P}{\lambda R \cos\beta} = 0 \qquad (7)$$

The calculation have shown that the convective part of the heat conduction , given by the drawing speed V_0 , supasses the diffusive part nearly completely so it can be neglected.
The final differential equation of temperature is given by relation 8.

$$\frac{\partial T}{\partial x}+\frac{2\alpha R}{c\rho V_0 R_0^2}T-\frac{2PR}{c\rho V_0 R_0^2}=0 \tag{8}$$

Using the mathematical program Maple 5 were solved equations 6 and 8. Figures 3 and 4 present the graphical results(the profiles of temperature, stress ,strain ,strain rate ,radius).In the numerical example which is presented in figures 5, 6 were assumed the following stages of the process: inductive heating between $x_0=0$ and $x_1=0{,}04$m, forced air cooling between $x_2= 0{,}06$m and $x_3= 0{,}1$m, free air cooling along the rest of the bar. The numerical parameters for simulation are the following:$P=10^8$ W/m^2, $R_0=2$ mm, $V_0= 0{,}0013$ m/s, $\sigma_0= 60$ N/mm^2, C=1196 N/mm^2, $\alpha_{1air}=30$W/m^{2}K,$\alpha_{2air}= 250$ W/m^{2}K, $\rho= 7850$ kg/m^3, c=660 J/kg K, $m_1=0{,}0025$, $m_2= -0{,}05877$, $m_3=0{,}1165$, $m_4=-0{,}0207$.Following data were obtained for steel OLC45.

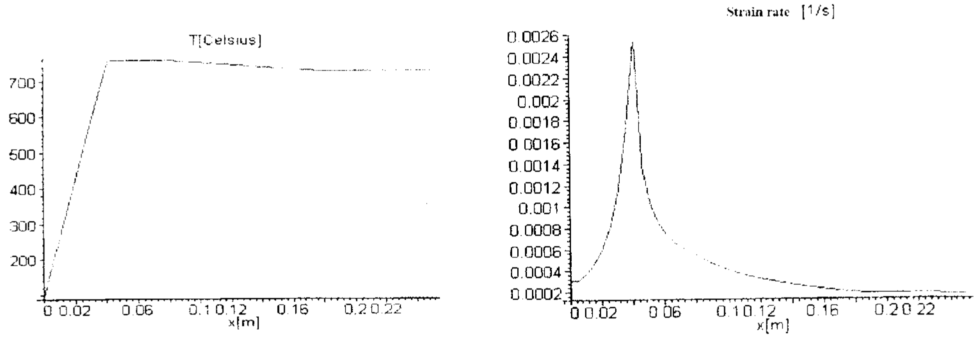

Figure 3 *Profiles of temperature and strain rate*

Figure 4 present the comparacy between theoretical an experimental profiles obtained for a wire with initial diameter of 4 mm.

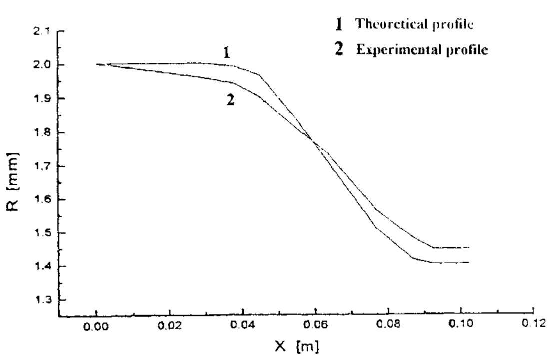

Figure 4 *Experimental and theoretical profiles*

For predicting the temperature distribution in dieless drawing were developed some theoretical models: finite element method, finite difference method, Bessel equations method.
For finite element method analysis, the model used is presented in figure 5[4].

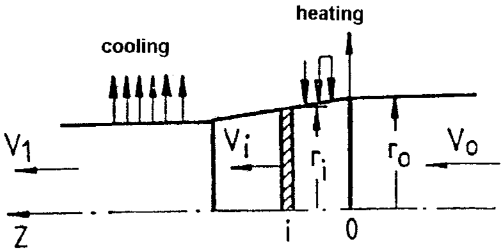

Figure 5 *The model for finite element analysis*

The fundamental differencial equation which includes the heat conduction, the heat input and the heat convections by material flow and the coolant is expressed by the following relation[5]:

$$\lambda \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + r\lambda \frac{\partial^2 T}{\partial r^2} - r \frac{\partial}{\partial z}(c\rho T V_z) = 0 \qquad (9)$$

where r, z are coordinates which express the radial and longitudinal directions, λ- thermal conductivity, T- temperature, c- specific heat, ρ- density, V_z- the longitudinal velocity of an element Figure 6 shows a comparison between the experimental and calculated values of the surface temperature when the drawing speed is $1,6 \times 10^{-2}$ m/s[5].

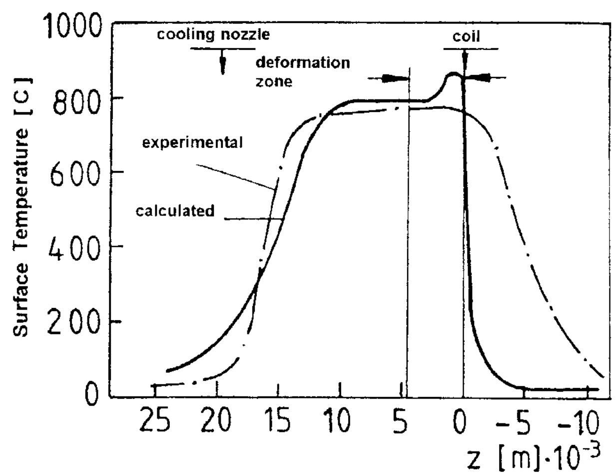

Figure 6 *Comparison between experimental and calculated surface temperature*

Figure 7 presentes the temperature distribution in the wire at a drawing speed of $1,6 \times 10^{-3}$ m/s[5]. One of the most important advantages of dieless drawing is the possibility of flexible change of the cross section of the product. The principle of cross section control is the constancy of volume which in the case of round bars can be expressed:

$$R_1 = R_0 \sqrt{\frac{V_0}{V_1}} \qquad (10)$$

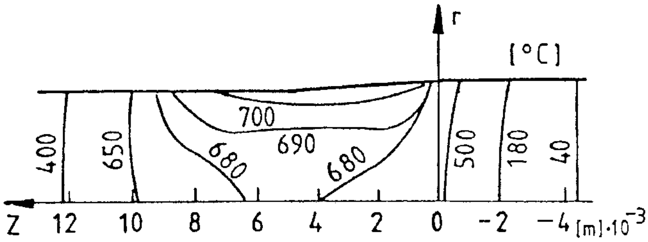

***Figure** 7 Calculated temperature*

The problem which has to be solved in this case is: for a desired special contour $R_1(x)$ the dependence of outgoing speed on time $V_1(t)$ is to be calculated. In the following we will give the example of producing a cone-contour where the variation of profile along the length l is:

$$R_1(x) = R_1(0) - \Delta R\frac{x}{l} \tag{11}$$

where $\Delta R = R_1(0) - R_1(l)$ is the radius variation on the length l

The drawing speed is this case is expressed by[3]:

$$V_1(x) = \frac{dx}{dt} = \frac{V_0 R_0^2}{R_1^2(x)} \tag{12}$$

By integration we find[3]:

$$t(x) = \frac{1}{V_0 R_0^2}\int_0^x R_1^2(\xi)d\xi \tag{13}$$

By elimination of x from egs. 12 and 13 we find $V_1(x)$.

3. CONCLUSIONS

The theoretical model which was presented in this paper had taken into account all parameters of the dieless drawing process, that permit to obtain two differential equations for profile and temperature variations during the process. Using the mathematical program Maple 5, were solved the differential equations and were represented some parameters of the process(profile, temperature, strain rate,). The researches demostrated the good agrements between the theoretical and experimental results.

4. REFERENCES

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